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$\boxed{\text{title:}}$ "Priority methods"

$N = \{0, 1, 2, \dots\}$

$f : N \to N$

f is computable

$\xrightarrow{\text{input } n} \boxed{M_f} \xrightarrow{\text{output}} f(n)$

$C \subseteq N$

C is computable (decidable) if χ_C is ~~computability~~ effective = computable.

$\xrightarrow{\text{input } n} \boxed{M_C}$ $\to$ Yes 1 if $n \in C$; $\to$ No 0 if $n \notin C$

(C codes decidable "problems")

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computably enumerable set $W \subseteq N$

There is a computable function f which enumerates it.

$W = \{f(0) = w_0, f(1) = w_1, f(2) = w_2, \dots\}$ ← search & find n

$\xrightarrow{\text{input } n} \boxed{TM}$ → output yes if $n \in W$; ⇢ never ends computes forever if $n \notin W$ ← no output computable $\Rightarrow$ computably enumerable

$\overset{?}{\Leftarrow}$

Partial characteristic function

If we have $f(0) = w_0 < f(1) = w_1 < f(2) < \dots$,

then W is computable.

(enumerated in an increasing order)

$W_0, W_1, W_2, \dots$ $W_e = \text{dom } P_e = \{x \mid P_e(x)\downarrow\}$

$= \{x \mid P_e(x) \text{ halts}\}$

$P_0, P_1, P_2, \dots$

most sets are non algorithmic

most sets are non computable

$\Rightarrow$ Halting Set.

$K = \{e : e \in W_e\} \subseteq N$ not computable

$P_e(e)\downarrow$

(halts)

Proof: Assume otherwise

then $\{e : e \notin W_e\}$ is computable

so computably enumerable $\{e : e \notin W_e\} = W_m$

$\boxed{m \in W_m \Leftrightarrow m \notin W_m}$

undecidability of the halting problem

$\{e : e \in W_e\} = HS$

$HS \leq X$

reduce algorithmically

X not computable

1956-57 The priority method

(Friedberg and Muchnik)

Computable structure with certain non-computable property.

There is a computable linear ordering of order type $\omega + \omega^*$ such that its ω-part is not computable (not even computably enumerable).

domain = N

$<$ is computable

$\underbrace{||||||}_{\omega}$ ... $\underbrace{||||||}_{\omega^*}$

$R = \omega$-part $\bar{R} = \omega^*$-part

1931 van der Waerden
Algebra field theoretic
algorithms is finite number
of steps.

0 4 2 2 4 4 6 ... 2 7 2 5 3 3 2 1

| | | | ... | | | |

0 2 4 6 ... 7 5 3 1

$R \neq W_0, W_1, W_2, \dots$

Simultaneous enumeration of all $W_0, W_1, \dots$

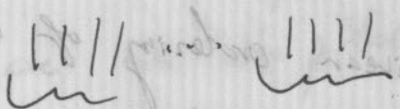
Goal:

At each stage s can put * single number in one of $W_0, W_1, W_2, \dots$

$W_e = \bigcup W_{e,s}$

$W_{e,0} = \emptyset \subseteq W_{e,1} \subseteq W_{e,2} \subseteq \dots$

Build a computable ordering in stages using $0, 1, 2, \dots$

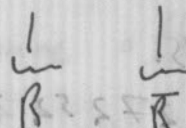


$\omega + \omega^*$ type

At each stage $\underline{\underline{s}}$

finite linear ordering

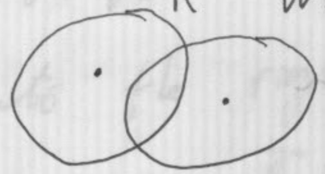
stage 0:



cannot change the ordering of elements to assure that the ordering is computable

$R \neq W_0, W_1, W_2, \dots$ Need a number

$R \neq W_e$ $c_e \in R - W_e$ or



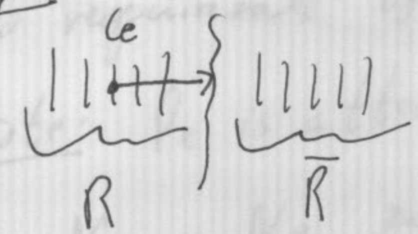
witness

$c_e \in W_e - R$

$c_e \notin W_{e,s}$ $c_e \in W_{e,s}$

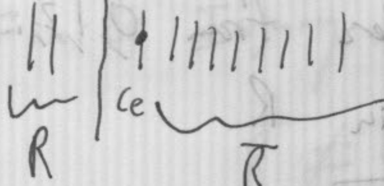
$c_e \in W_{e,s+s'}$

Stages:



$c_e \in W_{e,s} - R$

Requirements P_0, P_1, P_2



def Witness:

P_e: there is a number

$c_e \in W_e - R$ if W_e infinite.

$\boxed{W_e \neq R}$

Requirements

N_e: e^{th} number on R is never moved after a certain stage

? "~~do not move this number~~ after stage e"

Priority Listing

$N_0 > P_0 > N_1 > P_1 > N_2 > P_2 >$

$\overbrace{N_0 \; N_1}^{R}$ $\qquad \overbrace{\quad}^{\overline{R}}$

(1) (1) $\qquad$ | |

never change $\qquad$ $c_1 \; c_0$

Stage S:

$\underbrace{||||}_{R} \; \underbrace{||||}_{\overline{R}}$

Put the first two unused number from $0, 1, 2, \ldots$ into the leftmost empty spot in R and rightmost empty spot in $\overline{R}$.

Assume m is enumerated on $W_{e,s}$

If c_e has not been already defined, P_e requires attention

(||||) --- |||||

$N_0 \; N_1 \; N_2$

If m is not placed at $0^{th}, 1^{st}, 2^{nd}, \ldots,$ e^{th} place in R define $c_e \stackrel{def}{=} m$

and if m is in R, then move m AND all NUMBERS in R to the right of m into the rightmost places in $\overline{R}$

P_e is attacked at this stage

No requirements of higher priority injured.

Note: P_e is attacked at most once

$N_0, \ldots, N_e$ not injured by $P_e, P_{e+1}, \ldots$ Q_e

Finite Injury Priority Method

$$\overline{R} \neq W_e$$

$$d_e \in W_e - \overline{R} \qquad \square$$

Gerald Sacks infinite injury theory for infinite enumerable sets.